Club 44 F

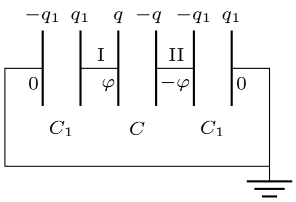
Edited by Elżbieta ZAWISTOWSKA

Solutions to problems from 3/2023

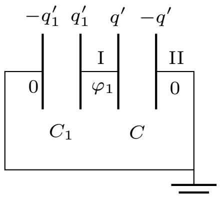
Let us recall problem statements:

754. The plates of a parallel plate capacitor with capacitance C are charged to potentials φ and $(-\varphi)$ relative to the ground. Each of the plates forms a capacitor with the ground, with capacitance C_1. Find the ratio of the electric field intensities between the plates of the capacitor with capacitance C initially and after grounding one of the plates.

755. A closed container is completely filled with water. Just above the bottom of the container, there is an air bubble. How will the pressure at the bottom change when the bubble rises to the surface?



Rys. 1



Rys. 2

754. The equivalent system to the one described in the problem is shown in Figure 1. A capacitor with capacitance C is charged with a charge of $q = 2\varphi C$, and the charges on the plates of capacitors with capacitances C_1 are $q_1 = \varphi C_1$. The total charge on the left plate is given by

$$Q = q + q_1 = (2C + C_1)\varphi. \tag{1}$$

After grounding the right plate, the equivalent system is shown in Figure 2. The charge on the plates of the capacitor C is $q' = \varphi_1 C$, where φ_1 represents the potential of the ungrounded plate. The charge on the capacitor with capacitance C_1 is $q_1' = \varphi_1 C_1$, and the total charge on the left plate is

$$Q' = q' + q_1' = \varphi(C + C_1) = Q. \tag{2}$$

Taking into account equation (1), we obtain

$$\varphi_1 = \frac{\varphi(2C + C_1)}{C + C_1}.$$

The electric field intensity between the capacitor plates is the ratio of the voltage to the distance between them. Therefore, the desired ratio of these intensities is

$$\frac{E}{E_1} = \frac{2\varphi}{\varphi_1} = \frac{2(C + C_1)}{2C + C_1}.$$

The rules of Club 44 F/M can be found on the webpage `deltami.edu.pl` (in Polish)

755. In the initial state, the air pressure inside the bubble is the same as the pressure of the water at the bottom of the container. The pressure difference between the bottom of the container and the top is given by $\Delta p = \rho g h$, where ρ is the density of water and h is the height of the container. During the ascent of the bubble, its volume remains unchanged because the liquid is practically incompressible. Therefore, the air pressure inside the bubble also remains constant. When the bubble reaches the top of the container, the pressure of the water under the surface is equal to the initial pressure at the bottom, so the pressure at the bottom has increased by Δp.

Club 44 M

Edited by Marcin E. KUCZMA

Solutions to problems from 3/2023

Let us recall problem statements:

857. Find all pairs of positive integers x, y such that $x^2 - 4y$ and $y^2 - 4x$ are squares of integers.

858. An equilateral triangle ABC of sidelength 1 and a segment DE of length 1 lie in the 3-space so that the segment has a point in common with the triangle. Show that one of the points A, B, C, D, E lies at a distance not exceeding 1 from each one of the other four points.

Problem 858 proposed by Michał Adamaszak of Copehagen.

857. Investigated are solutions of the system of equations

$$x^2 = a^2 + 4y, \qquad y^2 = b^2 + 4x \tag{1}$$

in integers $x, y \geqslant 1$, $a, b \geqslant 0$. Obviously, x and a must be of equal parity; (and likewise y and b).

By symmetry it will suffice to consider $x \geqslant y$. Then $x^2 = a^2 + 4y \leqslant a^2 + 4x$, i.e.

$$x^2 - 4x - a^2 \leqslant 0. \tag{2}$$

The first equation from (1) shows that $x^2 \geqslant a^2 + 4$, which (combined with the quadratic inequality (2)) yields the two-sided estimate

$$\sqrt{a^2+4} \leqslant x \leqslant 2 + \sqrt{a^2+4}. \tag{3}$$

It is easily seen that the number $a + 2$ lies in the interval $\left[\sqrt{a^2+4},\, 2 + \sqrt{a^2+4}\right]$ (of length 2). If $a \geqslant 1$, this is the only integer of the same parity as a (in this interval); so $x = a + 2$. The system (1) now forces $y = a + 1$ and $(a+1)^2 = b^2 + 4(a+2)$. Rewrite the last equation as $(a-1)^2 = b^2 + 8$; i.e.,

$$(a - 1 - b)(a - 1 + b) = 8,$$

with the unique solution (in integers) $a = 4$, $b = 1$. Hence